Optimization of bakery production by using branch and bound approach

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ABSTRACT
Mommy Ai Kitchen is one of the businesses specializing in the bakery business, producing cupcakes, birthday cakes, brownies, and donuts. However, it does not optimally determine each bakery’s production quantity, so it offers fewer profits and becomes a problem. This research aims to find the optimal production quantity so that this business maximizes profits. The method used was integer programming using the branch and bound approach, which counts the decision variable value using the simplex method. This research was based on the number of raw materials on hand—wheat flour, sugar, eggs, modal, and the profits of each bakery. Based on the analysis of the branch and bound approach, it was known that the maximum profit value was IDR 253,200, with eight alternative options for the bakeries that were produced. One of them was Mommy Ai Kitchen, which could produce three cupcakes, five birthday cakes, one brownie, and nine donuts to get that maximum profit. Meanwhile, Mommy Ai Kitchen’s estimation could produce one cupcake, one brownie, and six donuts using available materials with a profit of IDR 78,800. As a result, the profit difference between before and after integer programming was IDR 174,400.

Keywords: Bakery, Branch and bound, Integer programming, Optimization

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1. INTRODUCTION
Bakery products are instant foods that are popular with the public because they are more readily consumed and able to give adequate nutrients. Their taste, shape, and texture distinguish many bakery variants of products. The statistical data of the Ministry of Agriculture said that the consumption of fresh bread, sweet bread, and cookies had been increasing for five years (2014-2018). It reached the highest rise among the other foods, with a percentage rise of almost 500% for the last five years (2014-2018), followed by other bakery products, for example, cookies and other variants of bread. This case caused people’s interest in bakery products to increase continually, pushing the bakery business growth in Indonesia. Bakeries have run into significant growth in the last few years and are essential commodities today [1]. The COVID-19 pandemic has significantly affected all kinds of businesses, including economic businesses in Indonesia. Government policy regarding the restriction of people’s activities has an impact on the industry. Meanwhile, changing people’s eating patterns that make baking their primary interest and main food can save the bakery business [2], [3].

The bakery business tries to survive by focusing on developing the products and activities and obtaining profits optimally since its business has tough competitors in pandemic situations. Many competitors in the business mean merchants or businessmen must have the right strategy so that the aim of
the business goes well and obtain maximum profits [4]. Therefore, businessmen often face many problems in achieving their business goals. Usually, the problem is how to figure out the best number of products based on available resources in order to increase the maximum profit [5].

Mommy Ai Kitchen is one business that is still in the bakery business and is still running its business in the pandemic situation. It is located at Pasar Merah Street, Medan City, North Sumatra. Cupcakes, cookies, birthday cakes, and donuts are examples of bakery products. According to the data on the Industry Service’s site page in Medan City, there are 153 cake and bread businesses, both small and medium-sized businesses. Mommy Ai Kitchen is not the only bakery business; competition among bakery businesses has become more intense. Consequently, it must be able to solve every problem to survive and develop. One of the problems is that this business does not have a method to determine the production quantity of each bakery, so it can not obtain maximum profits. In addition, the raw materials of wheat flour, sugar, eggs, and capital become problems in the production process. Therefore, it is necessary to allocate the raw materials and modal so that it can maximize the profits.

One of the many methods that can help obtain maximum profits is linear programming. The linear programming method was the most recent method used during World War II [6]. The word “linear” is used to show the functions of mathematics in linear form. Meanwhile, programming is not about computer programming but specific mathematical techniques [7]. Linear programming is a mathematical technique to determine the solution to a problem that aims to maximize and minimize (optimization) something limited by limitations [8], [9]. There are three main elements to solve the production problem. They are (1) decision variable (it does not have a negative value to make it consistent with the real-life application), (2) objective function (maximum and minimum), and (3) constraint function (or the limitation of resources) that must have linear characteristics in linear programming [10]. According to Basrati [11] and Oladejo [12], the standard form of the linear programming result with the maximum objective function is,

\[ z = \sum_{j=1}^{n} c_j x_j \]  

with the constraints,

\[ \sum_{j=1}^{n} a_{ij} x_j \leq b_i \]  
\[ x_j \geq 0 \]

for and \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \)

Description,

\( z \) : Maximum objective function
\( c_j \) : Coefficient of the objective function
\( x_j \) : Decision variable
\( a_{ij} \) : Coefficient of the decision variable
\( b_i \) : Value of right side

The advantages of linear programming are that it can use many decision variables to achieve various possibilities for optimal resource utilization. The linear programming approach is applied to determine the feasibility of decision variables [13]. Linear programming can solve various problems, including finding the shortest part on the graph, maximizing network flow, transportation problems, task problems, portfolio optimization, dynamic programming, and integer programming [6], [14]. Integer programming is one of the linear programming languages where some or all decision variables must be non-negative integers [15]. All decision variables must be non-negative integers. These are called pure integer programming problems. All decision variables have a value of 0, meaning “no,” or 1, meaning “yes.” They are called binary integers. Some decision variables are required to be integers, and others are fractions. These are called mixed integer programming problems [16]. For example, Mommy Ai Kitchen’s problem is seen as pure integer programming because all production quantities of cupcakes, birthday cakes, brownies, and donuts require the result to be integers to achieve maximum profits. One of the problem-solving methods of integer programming is by using the branch and bound approach. The branch and bound approaches were found in Balas in 1977. It was a sequential solution that divided the space into subsets (certain branches) [17]. In addition, according to Taylor [18] branch and bound were a solution approach that divided feasible solution space into smaller subsets of solutions. They are applied repeatedly to form the search tree and the bounding process is applied by determining the optimal solution.
This research is based on the previous study by Megan Ryan and his friends entitled “Optimization of Pia Cake products by using integer programming method in small and medium enterprises’ XYZ in Waru Rejo Gempol Pasuruan Village.” The result of the research was integer programming method could be used to optimize the profits of two sizes of pia cake (small and big pia cakes) for each type of pia in small and medium enterprises’ XYZ [5]. In addition, Ammar and Emsimir used integer programming to optimize production planning in a company. Integer programming could optimize the profits of two kinds of products [19]. This research aims to optimize the production profits using integer-programming methods by forming linear programming with the profits of four bakeries’ products as its objective function.

2. METHOD

Bakery production data at Mommy Ai Kitchen in 2021 were used in this research. It is primary data. Primary data is collected by researchers directly from sources through interviews, questionnaires, calls, and others [20]. This primary data were collected by interviewing the owner of Mommy Ai Kitchen. This optimization uses a number from the beginning of data collection and analysis until the optimization result is collected so that the research approach is quantitative research. The method used in this research was the integer-programming model using the branch and bound approach, beginning with calculating the value of the decision variable using simplex methods. The implementation was done by using the quantitative method (QM) for Windows software.

2.1. Simplex method

Simplex is a technique for solving linear programming problems with many decision variables and constraints. George Dantzig developed the method in 1946. It provides a systematic way to check the solution point in the feasible part to determine the optimal value from the objective function. In this method, someone continues the steps from a basic feasible solution to others so that the objective function always increases [21]. The result of using the simplex method is that it collects better value to maximize profits [22].

2.2. Integer linear programming

Linear programming with the additional requirement of constraints to force the decision variables must have or become integer values [23]. One example is a variable that is defined as the number of people or objects, because the number of people or objects is definitely an integer, not and cannot be a fraction. Integer linear programming is a type of optimization problem where the variables are integer values and the objective function and equations are linear. According to Basriati [11], the general form of integer linear programming with the maximum objective function is,

$$\text{Maximum,} \quad z = \sum_{j=1}^{n} c_j x_j$$  \hfill (4)

Constraint,

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$  \hfill (5)

$$x_j \geq 0, \text{ integer for each } x_j$$  \hfill (6)

for and i = 1, 2, ..., m and j = 1, 2, ..., n

There are three types of integer linear programming models,

a. Pure integer programming is used if all decision variables are expected to have a non-negative integer value.
b. Binary integer programming is used if all decision variables have a value of 0, which means “no,” or 1 means “yes.”
c. Mixed integer programming is used if decision variables are expected to have integer values and contain fraction values.

2.3. Branch and bound approach

One of the common approaches used to solve constrained optimization problems is known as the branch and bound approach. This approach can solve the integer linear programming problem, where cost functions are based on relaxation for linear programming problems [24], [25]. The steps in solving integer programming by using the branch and bound approach are,
a. Solving the problems by using the simplex method without the integer constraints.
b. Checking its optimal solution. If the decision variable is an integer, then the optimal solution has been achieved. If it is not an integer, then continue to the third step. Choosing the variable with the huge fraction value (the enormous decimal number) from each variable to be branched into sub-problems.
c. Making the solution of step one as the upper limits and a solution of its decision variable that has been rounded.
d. Solving a linear programming model with the new constraints added to each sub-problem.
e. A feasible integer solution is as good as or better as the upper bound for each searched sub-problem. If this solution occurs, a sub-problem with the upper bound must be returned to step four.

The steps that are done in finishing this research as shown in Figure 1 are,
a. Interviewing the owner of Mommy Ai Kitchen to recognize and learn about their problems.
b. Data collection and problem identification. Collecting the data of the total raw material needed from each bakery product (cupcake, birthday cake, Brownies, and donuts) and the total available capital. Making integer programming modal in determining the number of bakery products and achieving the maximum profits are problem identification in this research.
c. Problem formulation and research purposes. The problem formulation in this research is how to make an integer-programming model that can represent the system that occurs. In addition, the research aims to determine the bakery product quantity optimally and obtain maximum profits.
d. Designing a program model. The model was designed after getting the information and the data. The model, which has been designed, will provide an abstraction with structural elements and operation between elements that form the system.
e. Running the program. A sequence of data processes does data processing by using QM for Windows software.
f. Analysis. Data collected and processed were analyzed to determine the number of production planning and the profits obtained later.
g. Conclusion: The content is the solution to the problem that is formulated in the problem formulation.

![Figure 1. Research method flowchart](image)

3. MODEL FORMULATION

The raw material composition of each production dough, the stock of raw materials, the modal, and the profits of each production data in a week are needed in this research. Data that can be collected from
Mommy Ai Kitchen business are shown in Table 1. Mommy Ai Kitchen does not have a method for determining the production quantity optimally in each kind of that bakery, so with those available materials, Mommy Ai Kitchen produces one cupcake, one birthday cake, and six donuts with Rp. 78.800 of profits in a week. Therefore, optimization by using an integer-programming model through the branch and bound approach to counting decision variable value by using the simplex method was analyzed in this research. Based on Table 1, Integer-programming modeling is done by (1) determining the decision variables, (2) creating an objective function, and (3) constraint formulations. This model is a step that needs to be done to get an optimal solution.

<table>
<thead>
<tr>
<th>Products</th>
<th>Raw Material Competitions per portion</th>
<th>Modals per Portion (IDR)</th>
<th>Profits per portion (IDR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wheat Flour (Gram)</td>
<td>Sugar (Gram)</td>
<td>Egg (Item)</td>
</tr>
<tr>
<td>Cupcake</td>
<td>95</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>Birthday Cake</td>
<td>175</td>
<td>150</td>
<td>6</td>
</tr>
<tr>
<td>Brownies</td>
<td>120</td>
<td>120</td>
<td>4</td>
</tr>
<tr>
<td>Donuts</td>
<td>150</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>Capacity</td>
<td>4.500</td>
<td>3.000</td>
<td>60</td>
</tr>
</tbody>
</table>

3.1. Integer programming application (QM for windows)

Next is to do data processing. The formed math model will be solved by using QM for Windows software with the integer-programming module. The steps in processing data are,

a. Inputting all data formulation (Objective function and constraints) into QM for windows software as shown in Figure 2.

b. Processing the data by clicking the solve button. Finally, display the solution from the result of the data input as shown in Figure 3.

Figure 3 shows the production result and the optimal profit with the limitation of the available resource (availability of raw materials and modal) in the Brach process in the 705th iteration. Finally, the result of the iteration is not feasible, and the value is less than the lower limit, so it cannot be branched anymore. Twenty-six branches/levels that are occurred in total. Besides that, based on Figure 3, the optimal profit obtained is Rp. 253,200 per week with several choices of bakeries that are produced per week. There are 8 choices, as shown in Table 2.
Optimization of bakery production by using branch and bound approach (Rahimullaily)

Figure 1. The formulation of data input

Figure 2. The solution software
Table 1. Constraints value with optimal profit IDR 253,200

<table>
<thead>
<tr>
<th>No</th>
<th>X₁</th>
<th>X₂</th>
<th>X₃</th>
<th>X₄</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>The 132&lt;sup&gt;nd&lt;/sup&gt; iteration, and the 11&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>2.</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>The 196&lt;sup&gt;th&lt;/sup&gt; iteration, and the 10&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>3.</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>14</td>
<td>The 293&lt;sup&gt;rd&lt;/sup&gt; iteration, and the 14&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>4.</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>9</td>
<td>The 311&lt;sup&gt;st&lt;/sup&gt; iteration, and 13&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>5.</td>
<td>11</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>The 362&lt;sup&gt;nd&lt;/sup&gt; iteration, and 14&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>6.</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>The 381&lt;sup&gt;st&lt;/sup&gt; iteration, and 17&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>7.</td>
<td>14</td>
<td>1</td>
<td>2</td>
<td>9</td>
<td>The 524&lt;sup&gt;th&lt;/sup&gt; iteration, and 20&lt;sup&gt;th&lt;/sup&gt; branch</td>
</tr>
<tr>
<td>8.</td>
<td>16</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>The 610&lt;sup&gt;th&lt;/sup&gt; iteration, and 22&lt;sup&gt;nd&lt;/sup&gt; branch</td>
</tr>
</tbody>
</table>

4. THE ANALYSIS OF THE BRANCH AND BOUND APPROACH

In the first step, solving the problem by using the simplex method without integer constraints obtained the solution of Z<sub>max</sub> = IDR 253,333.3 with x₁ = 0, x₂ = 7,451, x₃ = 0, and x₄ = 0. The value of x₄ from is not an integer, so the solution is invalid, so it becomes the branch into the sub-problem, and the profit value IDR 253,333.3 becomes the upper limit. With the down integer method x₁ = 0, x₂ = 7, x₃ = 0, and x₄ = 0 with the profit IDR 238,000, this result is feasible because both variables are integers. So, the profit value with the down integer becomes the button limit.

The first level or branch chooses the decision variable with a bigger fraction to do the branch based on the result of the previous decision variable, x₂, x₃ variable is branched to be sub-problem 1 and problem 2 with extra constraint for each sub-problem x₂ ≤ 7 and x₂ ≥ 8 so that is obtained the formation for sub-problem 1 and 2 as,

a. The solution to sub-problem: 1 x₁ = 1,2778, x₂ = 7, x₃ = 0, and x₄ = 0, with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

b. The solution to sub-problem 3 is infeasible

The second level or branch, choosing the decision variable that has a bigger fraction to do the branch base on the variable decision result in the previous sub-problem 1, is x₁ ≤ 1 and x₁ ≥ 2 so obtained the formulation for subproblems 1 and 2 as,

a. The solution to sub-problem 1: x₁ = 1, x₂ = 7, x₃ = 0, and x₄ = 0,6944 with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

b. The solution to sub-problem 2: x₁ = 2, x₂ = 6,7451, x₃ = 0, and x₄ = 0 with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

In the third level or branch, choosing the decision variable that has a bigger fraction to do the branch base on the result of x₁ = 1, x₂ = 7, x₃ = 0, x₄ = 0,694, and based on x₁ = 2, x₂ = 6,7451, x₃ = 0, x₄ = 0.

a. Choosing the decision variable with the bigger fraction to do the branch base on the result of x₁ = 1, x₂ = 7, x₃ = 0, x₄ = 0,694 is x₄. Variable x₄ is branched to be subproblems 1 and 2 with extra constraints for each sub-problems x₄ ≤ 0 and x₄ ≥ 1. So is obtained the formulation for sub-problem 1 and problem 2 as,

- The solution to sub-problem 1: x₁ = 1, x₂ = 7, x₃ = 0,833, and x₄ = 0 with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

- The solution to sub-problem 2: x₁ = 0,8778, x₂ = 7, x₃ = 0, and x₄ = 1 with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

b. Choosing the variable decision with the bigger fraction to do branch base on the result of x₁ = 2, x₂ = 6,7451, x₃ = 0, x₄ = 0 is variable x₂. Variable x₂ is branched to be sub-problem 1 and sub-problem 2 with extra constraints for each sub-problem x₂ ≤ 6 and x₂ ≥ 7. So is obtained the formulation for subproblems 1 and 2 as,

- The solution to sub-problem 1: x₁ = 4,111, x₂ = 6, x₃ = 0 and x₄ = 0 with Z<sub>max</sub> = IDR 253,333.3 (have not been integer and feasible)

- The solution to sub-problem 2 is infeasible.

The branch and bound above analysis can be drawn in the schema shown in Figure 4. Based on Figure 4, the process of the branch and bound is continued and will be finished if all design variables are integer and feasible. The problem solution uses QM for Windows, shown in Figure 4, so this branching process finishes at the 26<sup>th</sup> level/branch, and the total iteration calculation is as many as 705.
5. CONCLUSION

Mommy Ai Kitchen business has some constraints that are occurred in the production process, for example, the availability of wheat flour, sugar, egg, and capital. Therefore, production planning (to allocate the raw material and modal) and maximizing the business profit can be seen as an integer-programming problem. Determining the total of bakery products by using the branch and bound approach helps Mommy Ai Kitchen determine the production total in each bakery to obtain the maximum profit. Mommy Ai Kitchen will obtain the maximum profit of IDR 253,200 per week with some of the choices of the total of bakeries that are produced. First, 3 cupcakes, 5 birthday cakes, 1 brownie, and 9 donuts. Second, 5 cupcakes, 5 birthday cakes, 1 brownie, and 4 donuts. Third, 7 cupcakes, 3 birthday cakes, 0 brownies, and 14 donuts. Fourth, 9 cupcakes, 3 birthday cakes, 0 brownies, and 9 donuts. Fifth, 11 cupcakes, 3 birthday cakes, 0 brownies, and 4 donuts. Sixth, 10 cupcakes, 3 birthday cakes, 3 brownies, and 4 donuts. Seventh, 14 cupcakes, 1 birthday cake, 2 brownies, and 9 donuts. Eighth, 16 cupcakes, 1 birthday cake, 2 brownies, and 4 donuts. While the estimation of Mommy Ai Kitchen, with the available material, can produce 1 cupcake, 1 birthday cake, 1 brownie, and 6 donuts with a profit of IDR 78,800. So, the deviation of the previous profit from the profit after using the integer programming method is IDR 174,400.

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